Internationalizing the Mathematical Finance Course

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Abstract

About the year 2000, the Department of Mathematics and Computer Science, Albany State University (ASU), Albany, Georgia, USA envisioned the need to have a comprehensive curriculum revision based on recommendations of the Conference Boards of The Mathematical Sciences, the American Mathematical Society, the Mathematical Association of American, and based on the need to create attractive career pathways for our students in emerging fields and professions. Many Mathematics graduates were progressing to graduate schools in the fields of Applied Mathematics and Statistics. In subsequent years, our graduates started seeking jobs in the financial sector to become portfolio managers, Wall Street traders, bankers, insurers, and wealth fund managers. MATH 4330 Mathematics of Compound Interest course was created to give our students the opportunity to garner strong background to become confident future wealth managers. This course is inherently an attractive course to internationalize as economic growth is in the national interest of every nation, and the deep understanding of national and international financial institutions' functions is most essential. In this paper, I present the internationalization of Math 4330 Mathematics of Compound Interest, the associated outcomes, and the broader impact on students.

Keywords: internationalization, internationalizing curriculum; mathematics

Faculty members who enrolled to participate in internationalization of the curriculum were required to internationalize at least one course by developing the course syllabus, integrating course learning outcomes and course objectives reflecting anticipated deep and broad content knowledge of learners as well as incorporating international and intercultural dimensions in learning. Two courses which were internalized were MATH 1211-Calculus I and MATH 4330- Mathematics of Compounds Interest. In this paper, I will focus on the internationalization of MATH 4330-Mathematics of Compound Interest course. The case of MATH 1211 could be discussed analogously. The need to develop and teach MATH 4330- Mathematics of Compound

Interest course arose from students' interest in careers in the financial sector, as well as the financial sector's interest in recruiting and retaining young professionals with strong quantitative abilities. This course could be internationalized since every nation or society is endowed with continuous economic activities. A deeper understanding of global economic interactions and of economic relationships between countries is very important to students, and infusing international perspectives in this course has a wider impact on students enrolled in the course. Their career paths are widened, their skills and their competences enhanced, and their understanding of the world we live in deepened. It is essential to state here that a college-level course in economics was not a prerequisite for students to enroll in MATH 4330. Some of the economic concepts needed were covered in mathematical application problems in the College Algebra course and in Calculus I and Calculus II courses. Furthermore, the students who took this course were mostly Mathematics senior majors and therefore had the maturity and strong foundations to understand complex concepts.

Some students who took this course in the past had expressed interest in pursuing ***Actuarial Science*** and becoming actuaries. And this course, together with Mathematical Statistics (including probability theory) and Statistical Methods, gave them the content knowledge needed to prepare and successfully pass the foundational examinations of the Society of Actuaries (SOA) and the Casualty Actuarial Society (CAS). Membership in the above two named societies requires that a candidate pass a set of rigorous examinations, including Theory of Interest and Life Contingencies and Applied Mathematical Finance. Hence deep content knowledge and context were essential in this course.

Literature Review

Curricula and programs are usually influenced to a substantial extent by instructional faculty, as faculty reflect and revise program curricula depending on certain criteria, including student interest, job market, and the societal interest. The Mathematical Association of America (MAA) and the American Mathematical Society (AMS) have been advocating the infusion of the history of mathematics and ethno-mathematics in the mathematics undergraduate and graduate program curricula for more than fifty years. Many instructors teaching mathematics courses have in-depth knowledge of the history of mathematics and of its importance in the training of teachers

and future global leaders; they therefore integrate it in the courses they teach. As a matter of fact, The History of Mathematics course is a very popular course in our department.

During the civilizations of ancient Egypt, Mesopotamia, and Greece, these societies and cultures developed methods of doing trade and commerce. For example, in ancient Egypt, grain banks were developed, whereby farmers and traders stored their grains in such banks and paid a certain fee for such services. Some workers were paid for their work with bread and wine. There was trade by barter, too. However, about 500 BC, metal coins were developed and used as money. The people of ancient Greece developed even more sophisticated methods and tools for trade and commerce, creating measures and measuring instruments for olive oils and other products. They traded with the people of the Mediterranean world, developed ports and markets, and moved goods back and forth from the Greek City States to these trading partners (Katz 2004). For more than four thousand years, mathematics has played an immense role in strengthening societies, and even today, the economic and military capabilities of nations are directly proportional to their investments in education and the quality of the mathematics done in such countries. Many examples abound on the role of applications of mathematics in wealth creation of individuals and nations. To illustrate, Thales was a great philosopher and mathematician who lived in Miletus in ancient Greece (624-547 BC). He was the teacher of Euclid.

> *According to the story, he knew by his skill in the stars while it was yet winter that there would be a great harvest of olives in the coming year; so, having little money, he gave deposits for the use of all olive-presses in Chios and Miletus, which he hired at a low price. When the harvest-time came, and many wanted all at once and of sudden, he let them out at any rate he pleased. Thus, he showed the world that philosophers can easily be rich if they like, but their ambition is another sort, (Akyidirim and Soner, 2014).*

Goetzmann and Rouwenhorst (2005) present the following problem posed by Leornardo de Pisa (Fibonnaci) in his book ***Liber Abaci*** in 1202 A.D.:

> *A soldier is granted an annuity by the king of 300 bezants per year, paid in quarterly instalment of 75 bezarts. The king alters the payment schedule to an annual year-end payment of 300. The soldier can earn 2 bezarts on 100 per month (over each quarter) on his investment. How much is his effective compensation after the terms of the annuity changed*?

Similar examples from different cultures can be cited.

The advantages of internationalization of the curriculum are immeasurable. Colleges and universities have included in their university missions an intention to graduate global citizens who are aware and tolerant of other cultures, and at the same time marketable outside their geographical regions (Osakwe 2014). In recognition of the centrality of the importance of the global environment to the economic prosperity of the United States, in 2010 the Obama administration implemented a new initiative to support 100,000 US students to study and visit China within five years. Knight delineated four rationales for internationalizing the curriculum: economic, political, socio-cultural, and academic (2003). The American Council on Education (ACE) supports internationalization of the curriculum and highlights the fact that increasing understanding of other cultures, politics, economic growth, tolerance, acceptance, and peaceful coexistence with others, is essential for the modern global citizen (Olson, Green, and Hill, 2008).

There are certain concerns which should be handled with care due to obvious sensitivities, pedagogical concerns, time management issues, and quality and depth of content discussions. Some of these seemed to be in competition, yet the instructor must find a way to optimize course outcomes. In her paper. S. Staats (2015) discussed the Dilemma of Stereotypes, the Dilemma of Time Allocation, and the Dilemma of Assessments. According to her,

> *...teaching dilemma posed by internationalized college algebra is that studying data on economic and health inequalities can convey stereotypes of powerlessness and hopelessness among low-income people in other countries. This is particularly so among African countries, which*

predominate the extreme ranges of data sets on poverty and poor health outcomes for children and mothers.

On the dilemma of time allocation, S. Staats discussed the need to distribute course time carefully to cover as much course content as possible. She determined that in creating models with epidemiology data, social science issues and concepts arose, and that attracted student interest more than the mathematical content. This also led to the issue of assessment, thereby test questions about social ideas were included. The addition of this MATH course to the set of internationalized courses at Albany State University has contributed to the enrichment of the curriculum and has availed more students of the opportunity to garner course internationalization experience.

The rest of this paper is organized as follows: Section 3 of this paper focuses on methodology and learning activities, including the global attributes of the course. In Section 4, I focus on the course learning outcomes, and in Section 5, I present a typical problem viewed from international perspectives, and I provide a complete solution to the problem. In Section 6, I enumerate challenges encountered in the course. The conclusion and recommendations are discussed in Section 7. Following the References Page, I provide in the appendix some problems which could be of interest to the reader.

Methodology and Learning Activities

The instructional and learning activities reflected measurable outcomes and deliverables which were implicitly or explicitly inherent for this course.

Definitions and Examples

Students defined and discussed the following terms with examples: simple interest, compound interest, stock, bond, Coupon Bonds, Zero-Coupon Bonds, IRA, financial securities, US T-Bills, US T-Bonds, Mutual Funds, FDIC, annuities, future value of an annuity, present value of annuity, perpetuity, and IRA. Included in this discussion is the list of stock markets of the world and their net worth. There was an active class engagement. Financial topics, including currency exchange rates, were discussed.

Students did research on major currencies of the world and how currencies were traded in currency markets. Moreover, students discussed financial institutions in Europe, Asia, Africa, and the Americas, and they could observe that these institutions played common roles in various countries. The class also discussed the concept of Wealth. In general, people think of cash as wealth. But wealth is any object that has monetary value. Examples are land, houses, hotels, savings bonds, economic trees, equipment, farm products, and farms. Some of these are *Fixed Income Investments*.

Growth of Money

Using their mathematical foundations in calculus, students showed with many and varied examples why Compound Interest was preferred by banks and investors than Simple Interest. Nominal and effective rates of interest were discussed.

Solving Financial Mathematics Problems and Critical Thinking

Students learned how to formulate and solve many and varied common financial mathematics problems encountered everyday by a portfolio manager or wealth fund manager, whether they were sitting in New York, Tokyo, Beijing, Cairo, London, Buenos Aires, Paris, or Abuja.

Global Markets

The role of import and export markets and money exchanges were discussed. Export products (including commodities) of various countries and their valuations were discussed. Transportation, distances, geographical location, people and society, GDP, and other attributes were discussed as well.

Class Discussions and Group Presentations

Students were placed in groups of three to research about economic and financial activities (such as monetary policies, financial institutions, import and export products, and the GDP, rate, culture, religion, etc. of four countries such as Nigeria, Spain, China, and the US. Each group sought authentic information about the countries, including their geographical locations, people and culture, and they made class presentation thereafter.

The teacher summarized such discussions, explaining more formally the most essential economic activities that drove economic and wealth growth.

Learning Outcomes

The following topics in the course were discussed in the context of global issues: Simple interest, discount interest, compound interest, ordinary annuities, annuities certain, debt retirement methods, investing in stocks and bonds, depreciation and capital budgeting, future and present values of continuous streams, variable payment annuities, variable block of payments, stochastic payments, risk of default, and stochastic interest annuities, and topics in modeling and hedging. Prerequisites: MATH 2212-Calculus II with Analytic Geometry and MATH 211-Linear Algebra. Needed: Technology: TI 84-Plus Graphing Calculator, Microsoft Excel Solver, or any adequate Courseware.

Goal of the Course

Students to acquire deeper understanding of mathematics of finance and its global manifestations and they would be able to apply their knowledge as teachers, finance professionals and portfolio managers in the nation and the larger global environment.

Learning Outcomes

- Understood the applications of mathematics to the national and international financial markets.
- Discussed topics such as compound interest, annuities, stocks and bonds, and depreciation methods from the international perspectives.
- Created interest yielding investments using the concepts of Net Present Value, and Internal Rate of Return.
- Identified and explained the national and international financial instruments, economic activities, and productivity that create stability in the financial markets.
- Exemplified deep knowledge to become invaluable workers in the financial market where advanced mathematics is applied.
- Communicated understanding of concepts of mathematical of finance in oral and written forms.
- Applied technology to solve mathematical finance problems and interpreted results.

Specific Objectives

Upon completion of the course, students demonstrated with 80% mastery or higher the following knowledge-based competencies, and the professional skills and dispositions associated with each. Specifically, students:

1. Distinguished between simple interest and compound interest with examples.
2. Defined and discussed with international examples topics such as future value, present value, discount, nominal and effective rates, and compound rates.
3. Defined ordinary annuities, periodic payments, annuities certain, deferred annuities, forborne annuities, and perpetuities with varied examples.
4. Computed future and present values of annuities and interpreted the results.
5. Solved problems involving amortization, discount points owners' equity, sinking funds, and national and international real estate appreciation.
6. Created amortization tables, and design depreciation tables.
7. Delineated strategies for solving problems involving investing in national and international stocks and bonds, yield rates, and valuation of international stocks and bonds.
8. Performed capital flow analysis, and compute internal rate of return
9. Solved problems involving future and present values of continuous streams, inflation, and risk of default.
10. Summarized the role of international perspectives in the currency markets.
11. Discussed stock and bond markets in selected countries such as China, Nigeria, Ghana, Uganda, Tanzania, South Africa, India, Britain, France, and Spain.
12. Used global issues to discuss the mathematics of hedging and currency markets.
13. Solved simple problems involving basic stochastic calculus.
14. Developed and presented a project paper involving fixed income investments encountered in various countries of the world.

Solution to a class problem

We subsequently solved a typical mathematical problem with international dimensions through class discussion. This problem exemplifies internationalization of the course. Students could learn and discuss many attributes of international education using the problem stated in the sequel as a background. They could discuss that one could have an account in US Dollars in some Egyptian bank. Due to the durability and stability of the US Dollar, many rich people keep accounts in US Dollars. Moreover, US Dollars were accepted as the primary currency of international transactions. Students in this class also connected Egypt with History of Mathematics, religion, geography, the Arab World, Middle East, Africa, and Asia. Stability and lack of stability of several Middle East countries were also discussed. Here is a problem of significance.

Mousa was an Egyptian businessman. On February 11, 2001, his daughter Aisha gave birth to a baby boy. His name is Ahmed. Aisha also has five other children, two boys and three girls. She opened special bank accounts for each of her six children two months after their birth. On the day Ahmed was born, his grandfather, Mousa, opened a ten-thousand-Dollar bank account for him at a bank in Cairo. This account attracted quarterly compound interest. All accounts at this bank are in US Dollars. Mousa would like Ahmed to use this money to pay dowry on his wife at the age of 26. Furthermore, he would like the amount on this account to be $25,000 when Ahmed is 26 years.

(a) What is the annual interest rate if this account will have at least $25000 when Ahmed is 26 years old?
(b) What is the effective interest rate on this account?
(c) If Ahmed decides not to withdraw this money at 26, how much will he have in this account when he is 51 years old?

Solution:

(a) $P = \$10,000.00$ is the initial bank deposit the day Ahmed was born.

Suppose S is the future value of this account, the number of compounding periods is

$n = mt$ where m =4 is the number of compouding periods in a year and $t = 26$ is the number of years; $n = (4)(26) = 104.$

$$S = S_n = S_{104} = P(1 + \tfrac{i(m)}{m})^n = \$25000 = \$10000(1 + \tfrac{i(4)}{4})^{104}.$$

$$\$25000 = \$10000(1 + \tfrac{i(4)}{4})^{104}$$

$$(1 + \tfrac{i(4)}{4})^{104} = \frac{\$25000}{\$10000} = 2.5.$$ Hence we get that

$$1 + \tfrac{i(4)}{4} = \sqrt[104]{2.5} = 1.008849414 \Rightarrow \tfrac{i(4)}{4} = 0.008849414.$$

$$i(4) = 0.0353976576 \approx 3.54\%.$$

Hence the nominal interest rate is 3.54%.

(b) The effective rate on this account is

$$i_e = (1 + \tfrac{i\%(m)}{m})^m - 1 = (1 + \tfrac{3.54\%(4)}{4})^4 - 1 = (1.00885^4) - 1 = .035872714 \approx 3.59\%$$

(c) If Ahmed decides not to withdraw from this account until he is 51 years old, then the amount in that account will be

$$S_n = S_{204} = \$10000(1.00885^{204}) = \$60,342.88.$$

Other discussion topics such as, 'Dowry', followed.

The students and the instructor discussed the role of savings in paying dowry. This topic aroused many students' interest immensely.

Challenges

Several challenges were encountered in the course. The first challenge was how to optimally integrate attributes of internationalization in pedagogy. This challenge was reinforced by the fact that most of the students in the class were challenged by geographical knowledge, historical information, as well as lack of understanding of other cultures and traditions. Managing the time allocated to these courses to avail the students of the knowledge they need to understand the content was a challenge. The second

challenge emanated from the fact that some knowledge of economic terms was needed. Again, many students were not familiar with such knowledge. Those students must be accommodated in class activities. The third challenge was the depth and breadth of the content. The content was dense, hence more time was needed to cover all the topics. Despite the changes enumerated above, innovative pedagogical methods were used to enhance course outcomes: group assignments and presentations were allocated, and students who needed help were given individualized help by the instructor. Additional reading materials were distributed in class, and I made sure students could obtain credible information and resources on the internet. With my guidance, students developed research topics and presented their findings at an undergraduate research symposium in Atlanta.

Conclusions and recommendations

MATH 4330-Mathematics of Compound Interest could easily be internationalized because of its inherent global nature. Students who are trained to become portfolio managers, hedge fund managers, and wealth managers must have in-depth understanding of global economic activities, international monetary activities, international trade, geography, culture, and tradition. Understanding of currency markets for example required that the student knew who owned the currency, currency value fluctuations, devaluations, and currency conversions. The student should be deeply knowledgeable about international banks, brokerage firms, insurance companies, import and export trade organizations, transportation, and governmental and private institutions which dealt with wealth and growth of wealth. Furthermore, some of our students could secure jobs at the World Bank, International Monetary Fund (IMF), or with the US Foreign Service all over the world. The students who took this course understood global perspectives, and therefore have competitive advantage and greater opportunities to use their quantitative skills and abilities to work at many and varied international financial institutions and agencies. I recommend that more courses in Mathematics be internationalized as those courses would give students opportunities to learn both content and the global environment. The instructor must be prepared to carefully prepare a pedagogical path for his or her course to optimize outcome: a seamless integration of content and context is essential for the success of the course. Integrating research,

project, collaborative assignments, and class presentations is recommended. More importantly, the instructor should attend the workshops organized by the Office of Global Programs. I learned a lot by attending the workshops, and I continued to consult the director of the center and our external consultant for additional help when needed.

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Appendix

Here is a list of some typical problems encountered in this course. The names of the banks and investments are made up by the instructor as they have no real impact on the problems and their solutions.

1. Mr. Mark takes $4,750 loan from Capitol Bank of Georgia, USA, for 10 months that charges $330.42 interest. What is the interest rate for the loan?

2. Compute the maturity value of $42,571 note dated 9/24/15 with interest at 8.57% and a due date of 2/17/16. Use (i) Exact time and date, (ii) Banker's Rule.
3. What should you pay to buy a security that matures at $45,000 in 8 months if you wish to earn 11.5% on the investment?

4. On February 10, 2008, Mindi took a $25,000 loan which matures August 17, 2016 at 8.5% (4). Find the future value of the loan. (Draw a Time Line Diagram).

5. Larry invests $11,500 for five years at 7.25% interest. Find the amount if

   (a) Twice a year.
   (b) The money is compounded quarterly.
   (c) The money is compounded monthly.
   (d) The money is compounded daily (use 360 days/year).

6. If Carnegie Corporation of New York invested in an endowment of $135 million in 1911 at 4.52 %(1),
   (i) How much would have been in the funds in 1956, assuming that none of the interest was used during that time?
   (ii) If $42 million dollars is withdrawn every year after 1956, will this organization run out of money? If so, how long will it take for the organization to run out of money?

7. In 1956, Albany Investment Company (AIC) opened a savings account with $K earning a simple interest at an annual rate of 4.5%. Five years later, AIC closed the account and invested the accumulated amount in a savings account that yields 5.25% compound interest. Determine the number of years (since 1956) for the balance to reach $3K.

8. Mousa was an Egyptian business man. On February 11, 2001, his daughter Aisha gave birth to a baby boy. His name is Ahmed. Aisha also has five other children, two boys and three girls. She opened special bank accounts for each of her six children two months after their birth. On the day Ahmed was born, his grandfather Mousa opened a ten-thousand-Dollar bank account for him at a bank in Cairo. This account attracted quarterly compound interest. All accounts at this bank are in US Dollars. Mousa would like Ahmed to use this money to pay dowry on his wife at the age of 26. Furthermore, he would like the amount on this account to be $25,000 when Ahmed is 26 years.

   (i) What is the annual interest rate if this account will have at least $25000 when Ahmed is 26 years old?
   (ii) What is the effective interest rate on this account?
   (iii) If Ahmed decides not to withdraw this money at 26, how much will he have in this account when he is 51 years old?
   (iv) Perform a comparative analysis of interest rates and rate of returns in investments in Egypt, USA, China, and Nigeria.

9. A 273-day $2,000,000 Treasury Bill is bought with a bid of 93.12% Find the rate of return (ROR).
10. Ms. Thompson took a 30-year $265,000 mortgage loan from Albany Southwest Bank at 6.5% (12).

(a) Determine the minimum monthly payment for this loan.
(b) Find the future value of this annuity.
(c) Complete the first five rows of the amortization schedule below.

Payment No.	Payment	Interest paid	Principal Paid	Balance

0				$265,000
1				
2				
3				
4				
5				
6				

11. Amuzi Investment Bank (AIB) issues $126,000 loan to Dike Construction Company (DCC) at a nominal rate of 6.75 % (12). The terms of the loan are as follows: $60,000 will be paid back to AIB in 8 months, $50,000 will be paid back in 17 months, and $65,000 will be paid back in 24 months.
    (i) Find the Net Present Value (NPV).
    (ii) If AIB and DCC have an agreement that DCC will pay a nominal interest of 7.75% (12) if the loan is paid back in 3 years, how much will MCC pay back?
    (iii) What is the total interest of the loan if (ii) holds true?

12. At the age of 40 years, Mr. Rogers opened an IRA at Her Majesty's International Bank, London. He pays in $600 every month into an annuity account that yields a nominal interest rate of 9.5%. At the age of 50 years, he opens another IRA in which he pays in $750 per month at an annual nominal rate of 7.25%. At the age of 55 Mr. Rogers opens another account in which he pays in $625 every month, with this account having a nominal rate of 10.25%. All these accounts are maintained according to the terms until Mr. Rogers is 65 years.

    (a) How much will the future value of Mr. Rogers's retirement portfolio be when he is 62.5 years old?
    (b) Mr. Rogers plans to retire at 65 years. Suppose his IRA manager has promised him he would receive 6 % (12) interest on his retirement

portfolio. How much will Mr. Rogers have in this total annuity account when he turns 65 years old?

(c) Mr. Rogers plans to draw retirement amount one month after his 65th birthday. If he plans to receive $3410 per month after retirement, how long will his annuity last?

(d) Mr. Rogers is a sick man and would likely die when he is 80 years old. How much will he leave for his widow and children?